# An Automatic Control Solution for Impaired Airplane with Asymmetrical Engine Failures<sup>∗</sup>

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*Abstract*— An asymmetrical engine failure in a multi-engine aircraft during flight can pose an immediate threat and lead to a fatal crash if no appropriate corrective action is taken in time. Although almost all pilots are trained to follow a protocol to manually mitigate the crisis, asymmetrical engine failure still remains as one of the main precursors leading to fatal aircraft crash. In this paper, an automatic control solution designed based on  $H_2$  servomechanism control theory is proposed to effectively address this issue. The NASA GTM (Generic Transport Model) T2 aircraft, which is a twin-turbine unmanned aerial vehicle (UAV), is employed as a test bed. The UAV is 5.5% dynamically scaled to realistically simulate characteristics of a full-scale large civil transport jet aircraft.

### I. INTRODUCTION

According to the general aviation accident statistics [1], more than 395 accidents with multi-engine airplanes have been reported since Jan. 1996, and more than 3,600 people lost their lives during these accidents. A recent study in [2] also revealed that accidents in twin-engine aircraft carry a higher risk of fatality compared with single engine aircraft and constitute 9% of all general aviation accidents. A common sense would lead us to ask a question why the one with better engine redundancy is not performing better in critical flight safety.

There are two probable explanations. First, the probability for a twin-engine aircraft to have engine failure certainly is higher than that of a single-engine aircraft. In addition, the adverse effect due to the loss power of one engine is a direct cause leading to multi-engine aircraft accidents. A loss power of one engine for a twin-engine aircraft would immediately force the aircraft to yaw and roll towards the inoperative engine direction due to the asymmetrical thrust and unbalanced lift. Meanwhile, the aircraft would also pitch down and lose altitude because of the loss of approximately half of the total thrust. It is a crisis that demands immediate action to bring the aircraft to a stable feasible flight by powering up the operative engine to compensate the loss of thrust and provide enough airspeed, determining a new

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combination of rudder and aileron controls to counteract the adverse yaw and roll motions, and regulating the sideslip angle to zero [3].

Since the loss-of-control risk due to the asymmetrical engine failure is high, almost all the multi-engine aircraft pilots are required to received training in manually mitigating the problems following a protocol similar to one in [3]. As shown in the multi-engine accident statistics, the current practice of relying on manual mitigation for the critical flight safety issue is not a perfect solution. The mitigation procedure, which usually is aircraft model dependent, sometimes may be too complicated for pilots to implement correctly in time especially during a stressful emergency [1], [4].

In this paper, an automatic control solution designed based on *H*<sup>2</sup> control [5], [6], [7], [8] and servomechanism/regulator theory  $[9]$ ,  $[10]$ ,  $[11]$ ,  $[12]$ ,  $[13]$  is proposed to effectively address the loss-of-control issues caused by the asymmetrical engine failures. Our attempt is to make the mitigation process more reliable and easier for the pilot to implement. The NASA GTM (Generic Transport Model) T2 aircraft [14], a twin-turbine unmanned aerial vehicle (UAV), is employed as a test bed. The UAV is 5*.*5% dynamically scaled to realistically simulate characteristics of a full-scale large civil transport jet aircraft.

The remainder of the paper is organized as follows. In Section II, we briefly review the nomenclature of the GTM aircraft model and give the flight simulation diagram for later simulations. The diagram employs the full nonlinear untrimmed GTM model with practically constrained actuators. The controller design based on  $H_2$  servomechanism control theory is given in Section III. The simulations in Section IV demonstrate the effectiveness of the proposed control system in mitigating the asymmetrical engine failure and allow the impaired aircraft to fly normally with the remaining operative engine. Section V is the conclusion.

## II. PRELIMINARIES

## *A. The GTM Aircraft Flight Dynamics Model*

The NASA GTM (Generic Transport Model) nonlinear aircraft flight dynamics model, which is a 5*.*5% dynamically subscaled UAV to realistically simulate characteristics of a full-scale large civil transport jet aircraft, is used in this study as a test bed. The aircraft dynamics model is represented by the state space representation with the following equation;

$$
\dot{x}(t) = f(x(t), u(t))\tag{1}
$$

where the state vector is,

$$
x = \begin{bmatrix} V & \alpha & \beta & p & q & r & pN & pE & h & \phi & \theta & \psi \end{bmatrix}^T
$$
 (2)

and the control input vector that is,

$$
u = \left[ \begin{array}{cc} \delta_a & \delta_r & \delta_e & \delta_{TL} & \delta_{TR} \end{array} \right]^T \tag{3}
$$

The states and the inputs are represented as vectors in specific order, which will be kept through the entire study. The definition and units of the 12 state variables are *V*: total speed (knots), *α*: angle of attack (rad), *β*: side slip (rad), *p*: roll rate (rad/s), *q*: pitch rate (rad/s), *r*: yaw rate (rad/s), *pN*: position North (rad), *pE*: position East (rad), *h*: altitude (ft), *φ*: roll angle (rad), *θ*: pitch angle (rad), and *ψ*: yaw (rad). The definition and units of the 5 control inputs are  $\delta_a$ : aileron (rad),  $\delta_r$ : rudder, (rad),  $\delta_e$ : elevator (rad),  $\delta_{TL}$ : left throttle (%) and  $\delta_{TR}$ : right throttle (%). In GTM simulations, radian and radian/s are used as units. However, angles and angular rates will be displayed in degree instead of radian in our presentation for the ease of comprehension to the readers. The units for latitude and longitude positions are in radian in the GTM model, but they will be displayed as feet as well.

The GTM flight dynamics are described by 12 nonlinear equations, which are highly coupled. To design a linear controller for a nonlinear system, the nonlinear differential equations need to be linearized based on desired trim condition where the aircraft is at equilibrium. Due to the symmetrical structure of the GTM, the linearized 12 states flight dynamics can be decoupled to two set of flight dynamics: lateral and longitudinal flight dynamics.

For the straight level flight with a 4◦ angle of attack and  $0^\circ$  flight path angle, the following equation (Eq. 4) is the trim condition which is obtained by solving Eq. 1. Note that the throttles for the two identical jet engines are both set at 20*.*93% and the elevator input at 1*.*8◦ to keep a level flight with the specified angle of attack and total speed. The small nonzero aileron and rudder inputs are needed to cancel the gyro moment caused by engine rotations so that the aircraft can conduct a straight flight.

$$
x_{TrimA} = \begin{bmatrix} 83.67kt & 4^{\circ} & 0^{\circ} & 0^{\circ}/s & 0^{\circ}/s \\ 0^{\circ}/s & *ft & *ft & 2000ft & 0^{\circ} & 4^{\circ} & *^{\circ} \end{bmatrix}^{T}
$$
  
\n
$$
u_{TrimA} = \begin{bmatrix} -0.01^{\circ} & 0.01^{\circ} & 1.80^{\circ} \\ 20.93\% & 20.93\% \end{bmatrix}^{T}
$$
 (4)

This trim is called *T rim A* for later reference. Note that the values of the position state variables  $pN$ ,  $pE$ , the altitude *h* and the yaw angle  $\psi$  are left unspecified since they are irrelevant to the trim determination.

# *B. Control Simulation Using the Full Untrimmed GTM Aircraft with Actuator Constraints*

The closed-loop control system simulation will be conducted based on the switching control schematic diagram shown in Fig. 1. In this diagram, there are three possible ways to control the aircraft model. Pilot command allows pilots to manually control the aircraft actuators: engine throttle, aileron, elevator, and rudder. The second feedback



Fig. 1: Simulation Diagram.

controller is the baseline controller,  $K_B$ , which will be explained further in the following subsection. The baseline controller works fine in nominal condition for healthy aircraft. However, if one of the engines fails, the baseline controller will not be able to rescue the aircraft from loss-ofcontrol conditions since it has no ability to accommodate the engine failure. The third controller is the proposed accommodation controller  $K_A$  designed using the  $H_2$  servomechanism control theory. Although the controller is designed based on the linearized model at Trim A (Eq.4), with the regulation/tracking mechanism it works even when one of the two engines fails completely or partially. Comparison between a baseline controller and the accommodation controller will be presented in Section IV.

The GTM actuators' limits and constraints are given in Eq. 5. The left and right throttle,  $\delta_{TL}$  and  $\delta_{TR}$  respectively, are represented in the following by  $\delta_T$  since the two engines are identical.

$$
-20^{\circ} < \delta_a < 20^{\circ}, -30^{\circ} < \delta_r < 30^{\circ}, -30^{\circ} < \delta_e < 20^{\circ}
$$
\n
$$
0\% < \delta_T < 100\%, -300^{\circ}/s < \dot{\delta}_a < 300^{\circ}/s,
$$
\n
$$
-300^{\circ}/s < \dot{\delta}_r < 300^{\circ}/s, -300^{\circ}/s < \dot{\delta}_e < 300^{\circ}/s
$$
\n
$$
(5)
$$

# *C. Baseline Controller*  $K_B$

The structure of the baseline controller is shown in Fig. 2. This controller comes with the GTM package designed by NASA. It is designed to improve the aircraft stability and to reduce the pilot workload in flight operation. Inside of the baseline controller, there are three separate design tracks which are Alpha Controller (for pitch control), Roll Inner Loop (for roll control) and Yaw Inner Loop (for yaw control). Alpha controller defines the elevator control input position to track the desired angle of attack. Roll Innerloop design tunes the pilots aileron input to make the roll angle desired value. Yaw Innlerloop design regulates the pilot's rudder input to make the yaw rate zero. Baseline controller works fine for healthy aircraft, but it does not work when one of the two engines fails.



Fig. 2: Baseline Controller.

#### III. DESIGN OF ACCOMMODATION CONTROLLER

The design procedure of the Accommodation Controller *<sup>K</sup>*A in Fig. 1 will be given in the following. Note that the state vector  $x(t)$  and the control input vector  $u(t)$  in the GTM Nonlinear Aircraft Dynamics model are the actual state vector and the actual control input vector, respectively. But for the Accommodation controller  $K_A$  in Fig. 3, which is a linear controller designed for *Trim A*, the state vector and the control input vector are  $\bar{x}(t)$  and  $\bar{u}(t)$ , respectively. Their relationships with *x* and *u*, respectively, are described by the following equations,

$$
\bar{x} = x - x_{trim}, \quad \bar{u} = u - u_{trim} \tag{6}
$$

where  $x_{trim}$  and  $u_{trim}$  are the the values of state vector and control input vector at the trim. In other words, when  $x(t)$ and  $u(t)$  are at the trim, then  $\bar{x}(t)$  and  $\bar{u}(t)$  will both be zero vectors.

Due to the symmetrical structure of the aircraft, the flight dynamics can be decoupled into two parts: longitudinal and lateral dynamics. The longitudinal dynamics consists of 4 states: total speed *V*, angle of attack  $\alpha$ , pitch rate  $q$ , pitch angle  $\theta$ , and 3 control inputs which are elevator  $\delta_e$ , right and left throttles  $\delta_{TL}$  and  $\delta_{TR}$ . The state space representation of the linearized longitudinal dynamics model at *T rim A* can be found via Jacobian approach as follows,

$$
\begin{aligned}\n\dot{\bar{x}}_{Long} &= A_{Long}\bar{x}_{Long} + B_{Long}\bar{u}_{Long} \\
\bar{x}_{Long} &= \begin{bmatrix} \bar{V} & \bar{\alpha} & \bar{q} & \bar{\theta} \end{bmatrix}^T \\
\bar{u}_{Long} &= \begin{bmatrix} \bar{\delta}_e & \bar{\delta}_{TL} & \bar{\delta}_{TR} \end{bmatrix}\n\end{aligned} (7a)
$$

where

$$
A_{Long} = \begin{bmatrix} -0.0427 & 5.1918 & -0.0281 & -32.1740 \ -0.0032 & -2.8777 & 0.9554 & 0 \ -0.0053 & -43.4080 & -3.6111 & 0 \ 0 & 0 & 1 & 0 \ 0 & 0 & 1 & 0 \end{bmatrix}
$$
  
\n
$$
B_{Long} = \begin{bmatrix} -0.0445 & 0.0635 & 0.0636 \ -0.0048 & -0.0000 & -0.0000 \ -0.8285 & 0.0080 & 0.0080 \ 0 & 0 & 0 \end{bmatrix}
$$
 (7c)

Since the left and right engines may provide uneven thrusts to affect the lateral dynamics of the aircraft, the lateral dynamics model would have four control inputs: right and left throttle inputs,  $\delta_{TL}$  and  $\delta_{TR}$ , aileron,  $\delta_a$  and rudder  $\delta_r$ . Sideslip angle  $\beta$ , roll rate *p*, yaw rate *r* and roll angle  $\phi$  are the states in lateral dynamics. The state space representation of linearized lateral dynamics model at *T rim A* can be found via Jacobian approach as follows,

$$
\begin{aligned}\n\dot{\bar{x}}_{Lat} &= A_{Lat}\bar{x}_{Lat} + B_{Lat}\bar{u}_{Lat} \\
\bar{x}_{Lat} &= \begin{bmatrix} \bar{\beta} & \bar{p} & \bar{r} & \bar{\phi} \end{bmatrix}^T \\
\bar{u}_{Lat} &= \begin{bmatrix} \bar{\delta} & \bar{p} & \bar{r} & \bar{\phi} \end{bmatrix}^T \\
\bar{\delta}_a & \bar{\delta}_r & \bar{\delta}_{TL} & \bar{\delta}_{TR} \end{bmatrix}\n\end{aligned} \tag{8a}
$$

where

$$
A_{Lat} = \begin{bmatrix} -0.5840 & 0.0705 & -0.9856 & 0.2273 \\ -97.7031 & -6.5459 & 1.9994 & 0 \\ 33.5774 & -0.2805 & -1.4741 & 0 \\ 0 & 1 & 0.0699 & 0 \\ -1.0070 & 0.3093 & 0.0046 & -0.0050 \\ -0.0272 & -0.4765 & 0.0225 & -0.0223 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$
(8b)

The controller  $K_A$  will be designed to fly the aircraft at or near Trim A, as described in Eq.(4), to maintain a stable level flight and perform the tracking and regulation of flight path *<sup>γ</sup>*, altitude *<sup>h</sup>*, sideslip *<sup>β</sup>*, yaw angle rate *<sup>ψ</sup>*˙ and ground track (heading direction)  $\Psi$  in both straight flight and coordinated turn modes according to the pilot's commands. The structure of the accommodation controller  $K_A$  is depicted in Fig. 3. The controller is designed w.r.t. Trim A, which is a level straight flight with  $4^\circ$  angle of attack as described in Eq.(4).



Fig. 3: Accommodation Controller Design.

As seen from Fig. 3, the state vector  $x(t)$  of the untrimmed full nonlinear GTM aircraft flight dynamics model is available for feedback to the controller  $K_A$ . Since the controller is designed to fly the aircraft at or near Trim A,  $x_{trimA}$  needs to be subtracted from the actual state vector  $x(t)$  to obtain  $\bar{x}(t)$  as the input for the controller  $K_A$ . Although designed based on Trim A, which is a level straight flight with 4° angle of attack, the controller  $K_A$  is not restricted to only operate at Trim A flight condition. With the multivariable  $H_2$ servomechanism control structure, it can perform the tracking of altitude and flight path and the regulation of sideslip and yaw angle rate in straight flight and coordinated turn modes via the commands,  $\Delta h_{ref}$ ,  $\beta_{ref}$ , and  $\dot{\psi}_{ref}$  (or  $\Psi_{ref}$ ) given<br>but the nilet  $\Delta h$  is the little different of experiments for by the pilot.  $\Delta h_{ref}$  is the altitude differential commands for the altitude and flight path tracking in the straight flight or the coordinated turn modes. *<sup>β</sup>*ref and yaw angle rate reference  $\dot{\psi}_{ref}$  (or the desired heading direction  $\Psi_{ref}$ ) are employed<br>to achieve desired heading direction  $\Psi_{ref}$ ) are employed to achieve desired straight flight or coordinated turns. The output of the controller,  $\bar{u}(t)$ , needs to be added with  $u_{trimA}$ to obtain the actual control input  $u(t)$  for the untrimmed nonlinear GTM model.

Two optimal feedback controller gains are calculated by using LQR for longitudinal and lateral dynamics model.

The longitudinal and lateral state feedback gain matrices  $F_x$  and  $F_y$  are computed using the  $H_2$  optimization [5], [6], [7], [8] approach based on the state equations Eq.(7a) and Eq.(8a), respectively, as follows,

$$
F_x = \begin{bmatrix} -0.2504 & -14.7273 & 1.1882 & 18.4902 \\ -0.0858 & -0.7134 & 0.0783 & 1.1769 \\ -0.0859 & -0.7155 & 0.0785 & 1.1798 \end{bmatrix}
$$
 (9a)

$$
F_y = \begin{bmatrix} -73.6676 & 26.1092 & 5.8760 & 31.1152 \\ -30.6725 & -5.3574 & 13.7703 & -3.2873 \\ 2.5311 & -0.2289 & -0.7269 & -0.4221 \\ -2.5626 & 0.2405 & 0.7260 & 0.4346 \end{bmatrix}
$$
(9b)

The longitudinal regulator gains for flight path tracking are obtained as

$$
W_x = \begin{bmatrix} 83.6680 & -0.1194 & 0 & 0.8806 \end{bmatrix}^T
$$
 (10a)

$$
U_x = \begin{bmatrix} 10.6989 & 259.7092 & 259.7092 \end{bmatrix}^T \qquad (10b)
$$

by solving the following equations;

$$
A_{Long}W_x + B_{Long}U_x = 0
$$
  
\n
$$
C_{1u_x}W_x + D_{11u_x}U_x = 0
$$
\n(11)

where  $A_{Long}$  and  $B_{Long}$  are given in Eq. 7a and  $D_{11u_x} = 1$ and  $C_{1u_x} = \begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix}$  are chosen to achieve flight path tracking. To be able to track the altitude through the flight path angle  $\gamma$ , a gain constant  $R_{h\gamma}$  is used. The gain  $R_{h\gamma}$  is selected to have a reasonably fast tracking rate while not to cause large oscillations or actuator saturation. It is chosen to be

$$
R_{h\gamma} = 0.03^{\circ}/\text{ft} = 0.03 * (\pi/180) \text{ rad/s} \tag{12}
$$

Note that the reference input for  $\gamma$  tracking is

$$
\bar{\gamma}_{ref} = R_{h\gamma} (\Delta h_{ref} - \bar{h}) \tag{13}
$$

where  $\bar{h}$  is the altitude of the aircraft with respect to the altitude trim and  $\Delta h_{ref}$  is the desired altitude change.

The lateral regulator gains for side slip and yaw angle tracking are obtained as

$$
W_y = \left[ \begin{array}{ccc} 1 & 0 \\ 0 & -0.0699 \\ 0 & 1 \\ 1.3880 & 4.4061 \end{array} \right] \tag{14a}
$$

$$
U_y = \begin{bmatrix} -74.0808 & 1.4712 \\ 74.7022 & -3.1351 \\ 0.2307 & 1.3347 \\ 0.2263 & 1.3103 \end{bmatrix}
$$
 (14b)

by solving the following equations;

$$
A_{Lat}W_y + B_{Lat}U_y = 0
$$
  
\n
$$
C_{1u_y}W_y + D_{11u_y}U_y = 0
$$
\n(15)

where  $A_{Lat}$  and  $B_{Lat}$  are given in Eq. 8a and  $C_{1u_y}$  and  $D_{11u_y}$  are chosen as following;

$$
C_{1u_y} = \left[ \begin{array}{rrr} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \tag{16a}
$$

$$
D_{11u_y} = \begin{bmatrix} -1 & 0\\ 0 & -1 \end{bmatrix}
$$
 (16b)

The integrator gains  $c_\beta$  and  $c_\psi$  are chosen as 0.2 for side slip and yaw angle rate regulation. The heading direction yaw angle rate  $\Psi - \psi$  gain constant  $R_{\Psi \dot{\psi}}$  is used for converting a heading tracking control into a yaw angle rate tracking control. It is chosen to be

$$
R_{\Psi\dot{\psi}} = 0.3^{\circ}/\text{ft} = 0.3 * (\pi/180) \text{ rad/s}
$$
 (17)

# IV. SIMULATIONS

The results of two simulations will be presented in this section to demonstrate how the asymmetrical engine failure would affect the flight of a twin engine aircraft, and how a stabilizing tracking control system is designed to not only smoothly mitigate the effect of the failure, but also can continue conducting a normal flight using the remaining operative engine.

# *A. Simulation with Baseline Controller*

In the first simulation, during the first 20 seconds, the aircraft with the baseline controller was flying at the Trim A flight condition, which is a straight level flight with 4◦ angle of attack. Note that the thrusts of the engines were both at 2.25 lbf, with total thrust 4.5 lbf. But the thrust of the left engine started to decrease exponentially and down to zero at approximately 30 second mark. Since the baseline controller is designed to only control the three control surfaces: elevator, aileron, and rudder, the thrust of the right engine would remain unchanged throughout the simulation as shown in Fig.5. As expected, the aircraft would roll to the left to approximately  $-80^\circ$ , and yaw to the left continuously. At the same time, the altitude decreased 2000 ft in 45 seconds. The 3D plot at the bottom of Fig.4 showed that the aircraft was spiraling down until ir crashed to the ground.

#### *B. Simulation with Accommodation Controller*

The second simulation demonstrates the effectiveness of the accommodation controller in mitigating the asymmetrical engine failure.

Just like in the first simulation, during the first 20 seconds, the aircraft with the accommodation controller is flying at Trim A assuming no engine failure. From Fig. 8, the thrusts



Fig. 4: State Graphs of Simulation with Baseline Controller.



Fig. 5: Input Graphs of Simulation with Baseline Controller.

of the engines are both at 2.25 lbf, totally 4.5 lbf. At  $t = 20s$ , the left engine stopped working and the left thrust started to decrease quickly to zero in 10s, exactly the same as the first simulation. But the accommodation controller started to work to increase the right throttle in order to compensate the thrust loss from the left engine. Shortly after  $t = 150s$ , the right engine has reached 4.5 lbf of thrust. Meanwhile, the three control surfaces also started to work to balance the uneven lift right after the left engine failure.

It can be seen from Fig. 6 that initially the aircraft



Fig. 6: State Graphs of Simulation with Accommodation Controller.



Fig. 7: Position Graphs of Simulation with Accommodation Controller.



Fig. 8: Input Graphs of Simulation with Accommodation Controller.

lost about 130ft of altitude, but recovered back to 2000ft at t=150s. Since for the first 300s of the simulation, the controller is commanded to control the aircraft back to a straight level flight with the same initial 2000ft altitude. At t=300s, the aircraft reaches a new straight level flight equilibrium, although it is not the same as Trim A. The new equilibrium has a smaller angle of attack and a higher velocity. The state variables and the control inputs at t=300s are shown in the following.

$$
x(300) = [ 86.97 \text{kt} \quad 3.59^{\circ} \quad 0^{\circ} \quad 0^{\circ}/\text{s} \quad 0^{\circ}/\text{s} 0^{\circ}/\text{s} \quad * \text{ft} \quad * \text{ft} \quad 2000 \text{ft} \quad 2.02^{\circ} \quad 3.58^{\circ} \quad 90.13^{\circ} \quad ]^{T} u(300) = [ -0.85^{\circ} \quad -2.05^{\circ} \quad 2.06^{\circ} 0\% \quad 42.33\% ]^{T}
$$
 (18)

Considerable amount of aileron and rudder control inputs are used to eliminate the yaw and roll moments caused by thrust differential and keep the aircraft in straight flight condition. Fig. 7 shows that the ground track (heading direction)  $\Psi$ was kept at  $90^\circ$  up to t=300s. At t=300s, the controller is commanded to make a level coordinated turn with 3◦*/sec* yaw angle rate and zero side slip that would reverse the heading direction of the aircraft. That means the ground track angle  $\Psi$  will change from 90 $\degree$  to 270 $\degree$ . The trajectory of the aircraft during the simulation can be seen in Fig. 7.

Note that the trims at  $t=300s$  and at  $t=500s$  are identical before and after the coordinated turn. The state and control input values are given in Eq. 19 at  $t = 500s$ .

$$
x(500) = [ 86.97 \text{kt} \quad 3.59^{\circ} \quad 0^{\circ} \quad 0^{\circ}/\text{s} \quad 0^{\circ}/\text{s} 0^{\circ}/\text{s} \quad * \text{ft} \quad * \text{ft} \quad 2000 \text{ft} \quad 2.02^{\circ} \quad 3.58^{\circ} \quad 270.13^{\circ} \ ]^{T} u(500) = [ -0.85^{\circ} \quad -2.05^{\circ} \quad 2.06^{\circ} 0\% \quad 42.33\% ]^{T}
$$
 (19)

## V. CONCLUSION

The proposed multivariable  $H_2$  servomechanmism controller successfully solved the asymmetrical engine failure problem for the GTM aircraft. The controller can smoothly mitigate asymmetrical thrusts and uneven lift issues, maintain stability, and continue to provide versatile precision tracking capability for altitude, flight path, coordinated turn, and ground track angle even after one engine has completely failed. The same controller also provides the same stabilization and tracking performance. Therefore, if the same controller is employed as an auto pilot, it can continue to serve even when one of the two engines fails without the need to switch controllers.

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